Hypothesis Tests for Population Means

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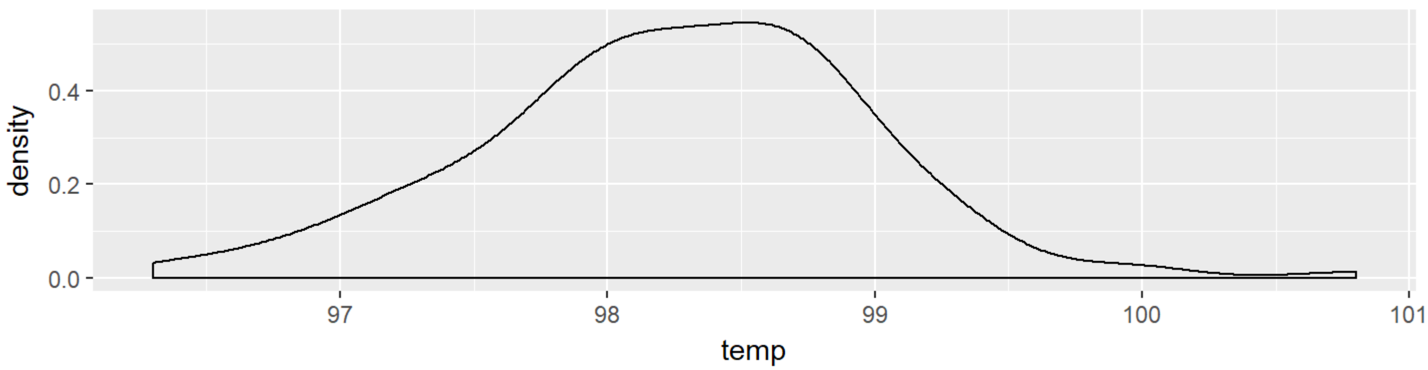
Outline of Hypothesis Tests (Again)

1. **Collect Data:** (For each of 8 attempts, was Paul's prediction right?)
2. Calculate a **test statistic**: $x = 8$ (observed number correct)
3. Write down **hypotheses**:
   - **Null Hypothesis**: Paul was just guessing: $p = 0.5$
   - **Alternative Hypothesis**: Paul is psychic: $p > 0.5$
4. **Sampling Distribution** of the test statistic, assuming null hypothesis is true.
5. **p-value**: probability of getting a test statistic at least as extreme as what we observed, assuming null hypothesis is true.
6. **Conclusion**: Compare the p-value to the significance level α. If the p-value is small, it's unlikely that Paul would get 8/8 right if he was just guessing, so we reject the null

Example: Body Temperatures

- It's generally believed that the average body temperature is 98.6 degrees Farenheit (37 degrees Celsius).
- Let's investigate with measurements of the temperatures of 130 adults.



- Hypotheses:
  - $H_0: \mu = 98.6$
  - $H_A: \mu \neq 98.6$
- What should our test statistic be?

A Key Result from Last Class

- $\bar{X} \sim \text{Normal}(\mu, \sigma/\sqrt{n})$
  - Across all samples, on average the sample mean is equal to the population mean μ.
  - The standard deviation of $\bar{X}$ is $\frac{1}{\sqrt{n}}$ as much as the standard deviation σ of values in the population.
- $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1)$
  - $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ is the distance of $\bar{X}$ from μ, in units of $SD(\bar{X})$.
- $\dfrac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$ (replace σ with its estimate, s).
  - $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ is the distance of $\bar{X}$ from μ, in units of $SE(\bar{X})$.

Test Statistic for a Mean

- Let's define our test statistic to be

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}, \text{ where}$$

μ_0 is the value of μ specified in H_0 (98.6 in this case)

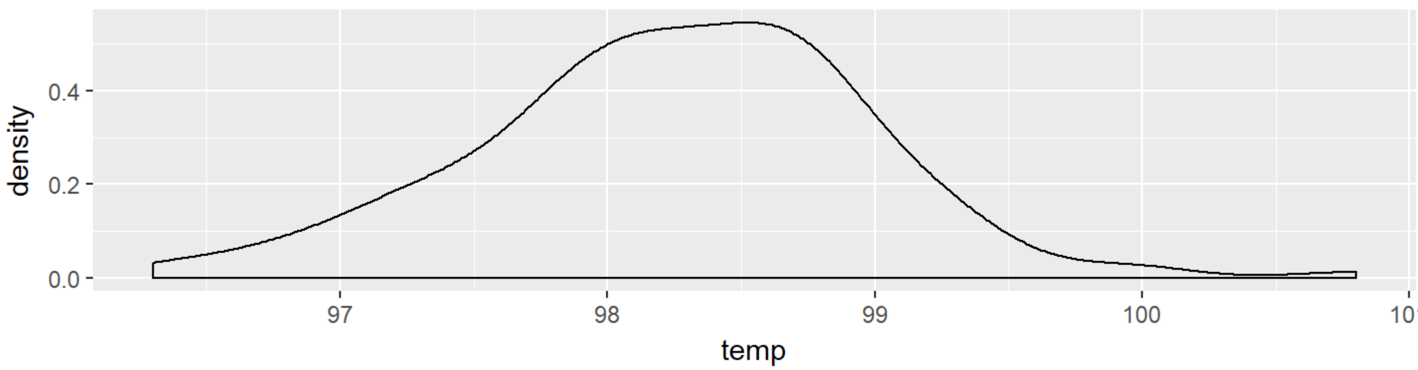
- How far was the sample mean from the hypothesized population mean, in units of our best guess at the standard deviation of $\bar{X}$?
- If the null hypothesis is true, then

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

Conditions to Check

- Observations are **independent**
- Population is **nearly normal** (unimodal, approximately symmetric)…
- …and **sample size** n is large enough (how big depends on how asymmetric distribution is)

Back to Body Temperatures



Assumptions for hypothesis tests about means:

- Independence
- Data distribution is nearly normal (unimodal and symmetric)
- Sufficient sample size

Hypotheses

- Null Hypothesis (H_0): $\mu = 98.6$ (where μ is the population mean temperature)
- Alternative Hypothesis (H_A): $\mu \neq 98.6$

Test Statistic

```
nrow(bodytemp)
```

```
## [1] 130
```

```
mean(bodytemp$temp)
```

```
## [1] 98.24923
```

```
sd(bodytemp$temp)
```

```
## [1] 0.7331832
```

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{98.249 - 98.6}{0.733/\sqrt{130}} = -5.460$$

Test Statistic in R

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

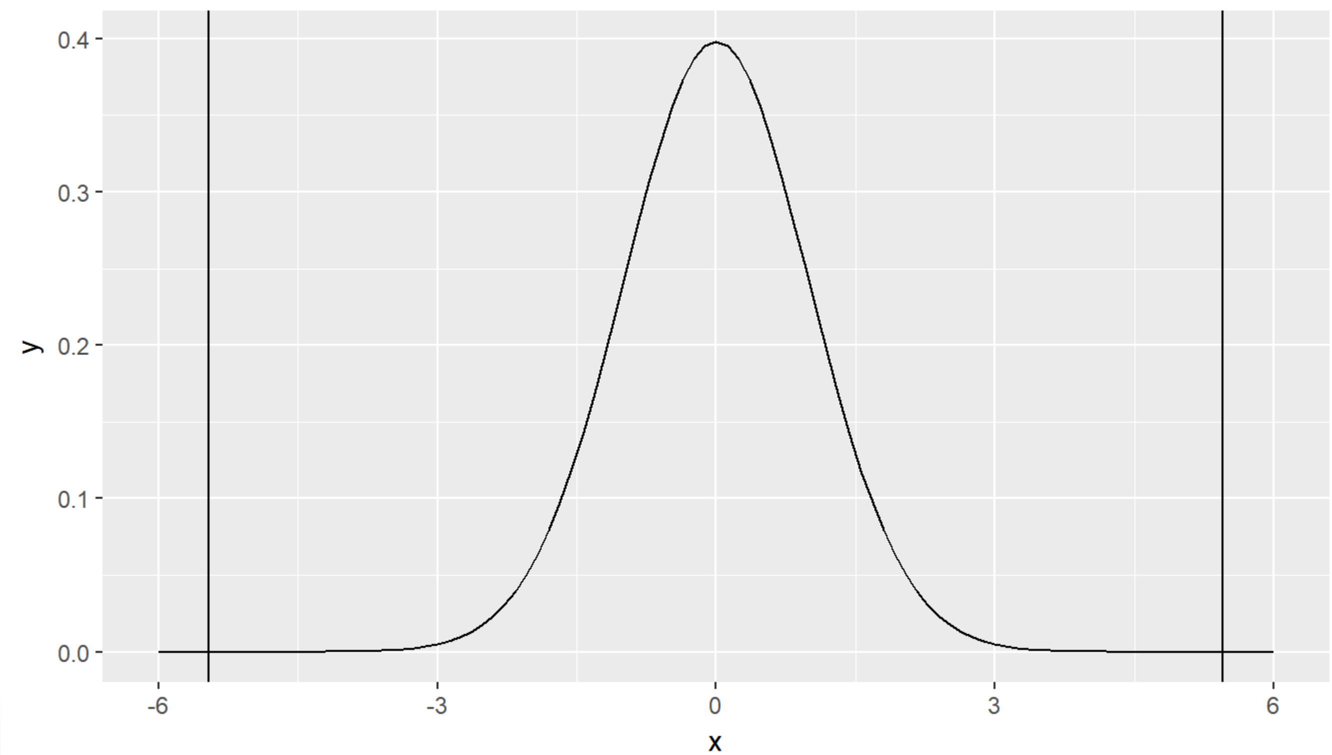
```
n <- nrow(bodytemp)
x_bar <- mean(bodytemp$temp)
s <- sd(bodytemp$temp)
mu_0 <- 98.6

t <- (x_bar - mu_0) / (s / sqrt(n))
t
```

```
## [1] -5.454823
```

P-value

- Probability of getting a test statistic at least as extreme as what we observed, assuming the null hypothesis was true.
- "At least as extreme" in either direction, since $H_A : \mu \neq 98.6$
- $t \sim t_{129}$ (since $n = 130$ and the degrees of freedom is $n - 1$)



Calculation of p-value

```
pt(-5.455, df = 129) # probability to the left of -5.455
```

```
## [1] 1.204343e-07
```

```
1 - pt(5.455, df = 129) # probability to the right of 5.455
```

```
## [1] 1.204343e-07
```

- Combined p-value is 0.000000241

Alternative Calculation in R

```
t.test(bodytemp$temp, mu = 98.6, alternative = "two.sided")
```

```
## 
##  One Sample t-test
## 
## data:  bodytemp$temp
## t = -5.4548, df = 129, p-value = 2.411e-07
## alternative hypothesis: true mean is not equal to 98.6
## 95 percent confidence interval:
##  98.12200 98.37646
## sample estimates:
## mean of x 
##  98.24923
```

Conclusion

- Compare the p-value to the significance level α. For example, if $\alpha = 0.001$ then

$$0.000000241 < 0.001, \text{so}$$

- The data provide enough evidence to conclude that the mean temperature is not 98.6 degrees F, at the $\alpha = 0.001$ significance level.

From Wikipedia

"The range for normal human body temperatures, taken orally, is 36.8 ± 0.5 °C (98.2 ± 0.9 °F). This means that any oral temperature between 36.3 and 37.3 °C (97.3 and 99.1 °F) is likely to be normal.

The normal human body temperature is often stated as 36.5-37.5 °C (97.7-99.5 °F). In adults a review of the literature has found a wider range of 33.2-38.2 °C (91.8-100.8 °F) for normal temperatures, depending on the gender and location measured."

- https://en.wikipedia.org/wiki/Human_body_temperature
- Never cite Wikipedia