Stat 140: Examples for Chapters 15 and 16

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Example 1

Background Refresher:

Let X and Y be random variables. It's always the case that

$E(X+Y) = E(X) + E(Y)$ and

If X and Y are *independent*, then it's also true that

$Var(X+Y) = Var(X) + Var(Y)$.

Note that it is *NOT* the case that $SD(X+Y) = SD(X) + SD(Y)$. This is one of the places where working with variances is more convenient than working with standard deviations.

The example (Number 15.37 from the book):

A grocery supplier believes that in a dozen eggs, the mean number of broken ones is 0.6 with a standard deviation of 0.5 eggs. You buy 3 dozen eggs without checking them. Assume that the three egg cartons you have selected are independent (maybe you bought them from 3 different stores).

(a) How many broken eggs do you expect to get?

Define X_1 = number of broken eggs in first carton. $E(X_1) = 0.6$, $SD(X_1) = 0.5$

X_2 = " " " " second " , $E(X_2) = 0.6$, $SD(X_2) = 0.5$

X_3 = " " " " " third " , $E(X_3) = 0.6$, $SD(X_2) = 0.5$

Total # of broken eggs is $X_1 + X_2 + X_3$.

$E(X_1+X_2+X_3) = E(X_1) + E(X_2) + E(X_3) = 0.6 + 0.6 + 0.6 = 1.8$ broken eggs.

(b) What's the standard deviation?

$SD(X_1+X_2+X_3) = \sqrt{Var(X_1+X_2+X_3)} = \sqrt{Var(X_1) + Var(X_2) + Var(X_3)}$

$= \sqrt{SD(X_1)^2 + SD(X_2)^2 + SD(X_3)^2} = \sqrt{0.5^2 + 0.5^2 + 0.5^2}$

$= \sqrt{3 \cdot 0.25} = 0.866$

Example 2

Background Refresher:

The binomial distribution can be used to model **the number of successes in n trials**, when the trials are *independent* and each has *success probability* p.

We're never going to calculate probabilities involving Binomial distributions by hand, but it's good to be generally aware that there is a formula for the number of successes that we could derive if we wanted to. If $X \sim$ Binomial(n, p), then

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}.$$

We *will* need to know that $E(X) = np$ and $Var(X) = np(1-p)$.

The example (adapted from number 16.38 in the book)

A wildlife biologist examines frogs for a genetic trait he suspects may be linked to sensitivity to industrial toxins in the environment. Previous research had established that this trait is usually found in 1 of every 8 frogs. He collects and examines 12 frogs.

(a) Recall that Bernoulli trials satisfy three conditions: (i) there are only two possible outcomes for each trial; (ii) the probability of each of those two outcomes is the same across all of the trials; and (iii) the trials are independent. If the biologist took the sample of frogs from one pond that has been exposed to toxins where he thinks there are a total of about 60 frogs, would it be appropriate to model the results as Bernoulli trials? Discuss each of the three conditions for Bernoulli trials.

(i) two outcomes: each frog either has or doesn't have the genetic trait
(ii) it is reasonable to assume that each frog we sample has the same chance/probability of having the genetic trait.
(iii) if the selection of the frogs in the sample was done carefully, and we could be sure that they were not related, or taken from the same area, etc., it might be appropriate to assume that the frogs were independent.

Depending on how carefully the sampling of frogs was done, a Binomial model might be appropriate

(b) Regardless of your answer to part (a), let's model the number of frogs in the sample with a binomial distribution. If the frequency of the trait has not changed, what are the expected value and standard deviation of X, the number of frogs in the sample with the trait?

If we model $X \sim \text{Binomial}(12, 0.125)$ (since $1/8 = 0.125$), then

$E(X) = n \cdot p = 12 \cdot 0.125 = 1.5$

$SD(X) = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{12 \cdot (0.125) \cdot (0.875)}$
$= 1.15$

(c) What's the probability that exactly 42 of the frogs in the sample have the trait? Use the `dbinom` function in R.

I meant 12... there are only 12 frogs in the sample!

dbinom(x=12, size=12, prob=0.125) → output is $\boxed{1.46 \times 10^{-11}}$

$P(X=12) = 1.46 \times 10^{-11}$

(d) What's the probability that 10 or fewer of the frogs in the sample have the trait? Use the `pbinom` function in R.

pbinom(q=10, size=12, prob=0.125)
output is 0.999....

$P(X \leq 10) = 0.999...$