Stat 140 Probability Examples

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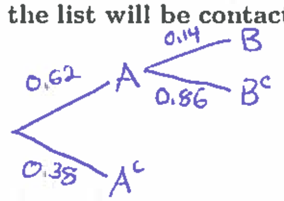
Polling

Opinion polling organizations contact their respondents by sampling random telephone numbers. According to the Pew Research Center, as of 2012, interviewers were able to reach about 62% of all US households. Among those contacted, about 14% agreed to participate in surveys.

1. What is the probability that the next household on the list will be contacted but will refuse to cooperate?



Define the events
A = we reach the next household, and
B = they agree to participate.
We are given $P(A) = 0.62$ and $P(B|A) = 0.14$

P(we reach the next household and they refuse to participate) $= P(A \text{ and } B^c)$
$= P(A) \cdot P(B^c|A) = 0.62 \cdot 0.86 = \boxed{0.5332}$

2. What is the probability that for the next household on the list, the interviewer will either not be able to contact the household, or they will contact the household but noone in the household will agree to participate in the survey?

$P(A^c \text{ or } (A \text{ and } B^c)) = P(A^c) + P(A \text{ and } B^c)$ (since A^c is disjoint from (A and B^c))

$= 0.38 + 0.533$

$\boxed{= 0.913}$

3. What is the probability that the next household on the list will agree to participate in the survey?

$P(A \text{ and } B) = P(A) \cdot P(B|A) = 0.62 \cdot 0.14 = \boxed{0.087}$

Blood Types

The American Red Cross says that about 45% of the US Population has Type O blood, 40% has Type A blood, 11% has Type B bloo~~k~~d, and the rest have Type AB bloo~~k~~d.

1. Someone volunteers to give blood. What is the probability that this donor

(a) Has Type AB blood?

$$\begin{aligned} P(\text{Type AB}) &= 1 - P(\text{Type O or Type A or Type B}) \\ &= 1 - [P(\text{Type O}) + P(\text{Type A}) + P(\text{Type B})] \\ &= 1 - [0.45 + 0.40 + 0.11] \\ &= 1 - 0.96 \\ &= \boxed{0.04} \end{aligned}$$

(b) Has Type A or Type B blood?

$$\begin{aligned} P(\text{Type A or Type B}) &= P(\text{Type A}) + P(\text{Type B}) \\ &= 0.4 + 0.11 \\ &= \boxed{0.51} \end{aligned}$$

2. Among four potential blood donors, what is the probability that

(a) All are Type O? Let's assume that the blood types of the four donors are <u>independent</u>.

$$\begin{aligned} &P(\text{Donor 1 Type O and Donor 2 Type O and Donor 3 Type O and Donor 4 Type O}) \\ &= P(\text{Donor 1 Type O}) \cdot P(\text{Donor 2 Type O}) \cdot P(\text{Donor 3 Type O}) \cdot P(\text{Donor 4 Type O}) \\ &= 0.45 \cdot 0.45 \cdot 0.45 \cdot 0.45 \\ &= \boxed{0.041} \end{aligned}$$

(b) No one is Type AB?

$$\begin{aligned} &P(\text{Donor 1 not Type AB and Donor 2 not Type AB and Donor 3 not Type AB and Donor 4 not Type AB}) \\ &= P(\text{Donor 1 not Type AB}) \cdot P(\text{Donor 2 not Type AB}) \cdot P(\text{Donor 3 not Type AB}) \cdot P(\text{Donor 4 not Type AB}) \\ &= [1 - P(\text{Donor 1 Type AB})][1 - P(\text{Donor 2 Type AB})] \cdot [1 - P(\text{Donor 3 Type AB})] \cdot [1 - P(\text{Donor 4 Type AB})] \\ &= [1 - 0.04][1 - 0.04][1 - 0.04][1 - 0.04] \quad \text{from part 1. (a)} \\ &= \boxed{0.85} \end{aligned}$$

(c) They are not all Type A? We could do this the same way I did part (b), or...

$$\begin{aligned} P(\text{not all Type A}) &= 1 - P(\text{all Type A}) \\ &= 1 - P(\text{Donor 1 Type A}) \cdot P(\text{Donor 2 Type A}) \cdot P(\text{Donor 3 Type A}) \cdot P(\text{Donor 4 Type A}) \\ &= 1 - 0.4 \cdot 0.4 \cdot 0.4 \cdot 0.4 \\ &= \boxed{0.97} \end{aligned}$$