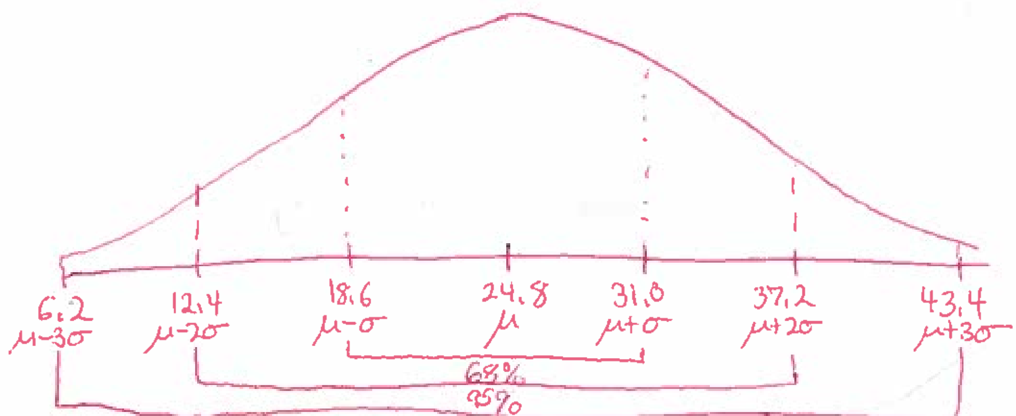
Solutions

Stat 140 - More Practice with Normal Distributions

(Adapted from Exercise 5.7 in the book) Environmental Protection Agency fuel economy estimates for automobile models tested recently predicted a mean of 24.8 mpg and a standard deviation of 6.2 mpg for highway driving. Assume that a normal model can be applied.

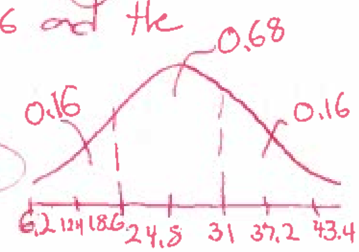
There are two sides.

(a) Draw the model for auto fuel economy. Label it, showing what the 68-95-99.7 rule predicts.



(b) About what percent of autos get more than 31 mpg?

- The area between 18.6 and 31.0 is 0.68.
- The remaining area of (1−0.68=0.32) is split evenly between the two tails, so the area to the left of 18.6 is 0.16 and the area to the right of 31.0 is 0.16.
- Therefore, 16% of autos get more than 31 mpg.



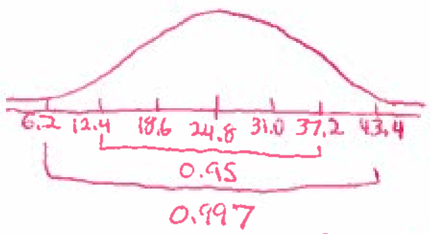
(c) About what percent of cars get between 31 and 37.2 mpg?

if this is not clear, draw a picture to convince yourself!

- The proportion of cars that get between 31 and 37.2 mpg can be calculated as the difference between (proportion less than 37.2) and (proportion less than 31)
- From part (b), we know that the area between 18.6 and 31 is 0.68, and the area ~~between~~ to the left of 18.6 is 0.16.
- That means the total area to the left of 31 is 0.68+0.16=0.84.
- Using the 68-95-99.7 rule, the area between 12.4 and 37.2 is 0.95.
- The remaining area of (1−0.95)=0.05 is split evenly between the two tails, so the area to the left of 12.4 is $\frac{0.05}{2}=0.025$.
- That means the total area to the left of 37.2 is 0.95+0.025 =0.975
- Therefore, the proportion of cars getting between 31 and 37.2 mpg is 0.975−0.84, which is 0.135, or 13.5%.

(d) About what percent of cars get less than 10 mpg? (You could do this using either R, or by getting approximate bounds on the probability from the 68-95-99.7 rule – it's good to know how to do it both ways)

Method 1: 68-95-99.7 Rule:



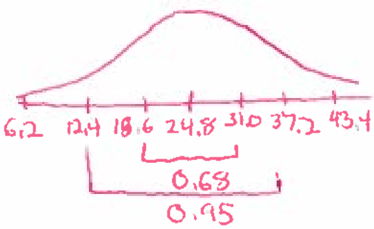
The area to the left of 12.4 is $\frac{1-0.95}{2} = 0.025$, so 12.4 is the 2.5th percentile.
The area to the left of 6.2 is $\frac{1-0.997}{2} = 0.003$ so 6.2 is the 0.3rd percentile

Somewhere between 0.3% and 2.5% of cars get less than 10 mpg.

Method 2: R. Use the pnorm function.
The command is: pnorm(q=10, mean=24.8, sd=6.2)
The output I got when I typed this into the console was 0.00849161.
So about 0.8% of cars get less than 10 mpg.

(e) What is the 90th percentile of fuel economy? (You could do this using either R, or by getting approximate bounds on the percentile from the 68-95-99.7 rule – it's good to know how to do it both ways)

Method 1: 68-95-99.7 Rule:

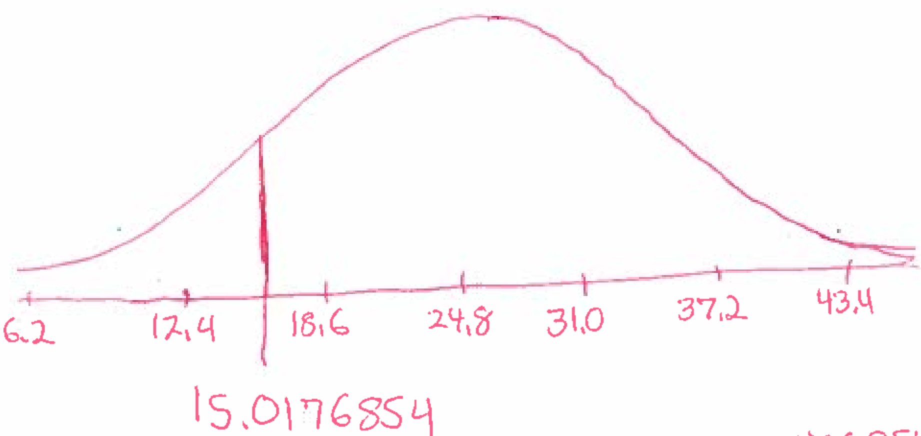


- The area to the left of 18.6 is $\frac{1-0.68}{2} = 0.16$, so the total area to the left of 31 is $0.68 + 0.16 = 0.84$.
- 31 is the 84th percentile of fuel economy.
- The area to the left of 12.4 is $\frac{1-0.95}{2} = 0.025$, so the total area to the left of 37.2 is $0.95 + 0.025 = 0.975$.
- 37.2 is the 97.5th percentile of fuel economy.

So the 90th percentile of fuel economy is between 31 and 37.2.

Method 2: R: qnorm(p=0.9, mean=24.8, sd=6.2) gives output of 32.7. The 90th percentile of fuel economy is 32.7 mpg

(f) What percent of cars get exactly 15.0176854 mpg? (Draw a picture, and remember that we interpret probabilities as areas under the curve)



The area under the curve at 15.0176854 MPG is 0!
According to the normal model, no cars get exactly 15.0176854 mpg. What if there was a car in our sample that got exactly 15.0176854 mpg? The normal model would not reflect that?