Stat 140 - More Probability Exercises

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AIDS Testing

The ELISA test for AIDS is used in the screening of blood donations. As with most medical diagnostic tests, the ELISA test is not infallible. If a person actually carries the AIDS virus, experts estimate that this test gives a positive result 97.7% of the time. (This number is called the *sensitivity* of the test.) If a person does not carry the AIDS virus, ELISA gives a negative result 92.6% of the time (the *specificity* of the test). Recent estimates are that 0.5% of the American public carries the AIDS virus (the *base rate* with the disease).

1. When the American Red Cross receives blood donations, they test them for HIV/AIDs. Suppose a blood donation from a particular person has tested positive. Given this information, how likely do you think it is that the person actually carries the AIDS virus? Don't do any calculations for now, just make a guess.

SOLUTION:

Let's define some relevant events:

A = the event that a (randomly selected) person carries the AIDs virus B = the event that a (randomly selected) person tests positive for AIDs

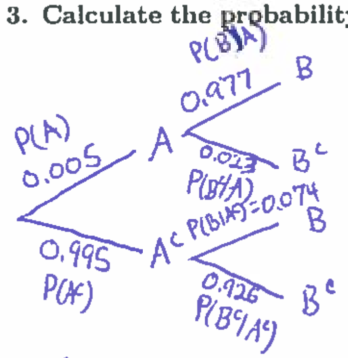
2. From the description above, determine the following (note that there are no calculations to do here - just matching up numbers in the text to probabilities of events):

$P(A) =$ 0.005

$P(B|A) =$ 0.977

$P(B^c|A^c) =$ 0.926

3. Calculate the probability that someone who has tested positive for AIDs actually has AIDs.



At each branch of the tree, the probabilities have to sum to 1.

Bayes' Rule says that

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A^c) \cdot P(B|A^c)}$$

$$= \frac{0.005 \cdot 0.977}{0.005 \cdot 0.977 + 0.995 \cdot 0.074}$$

$$= 0.062$$

Email Spam

According to Wikipedia, about 80% of all email that is sent is spam (https://en.wikipedia.org/wiki/Email_spam). According to this article in Wired magazine, as of 2015, Google's spam filter for Gmail correctly identifies 99.9% of all spam emails as spam, and only incorrectly identifies 0.05% of non-spam emails as spam (https://www.wired.com/2015/07/google-says-ai-catches-99-9-percent-gmail-spam/).

Define A to be the event that an email is a spam message, and B to be the event that an email is identified as spam by Gmail's filters.

1. From the description above, determine the following (note that there are no calculations to do here - just matching up numbers in the text to probabilities of events):

$P(A) = 0.8$ $\Rightarrow P(A^c) = 0.2$

$P(B|A) = 0.999$ $\Rightarrow P(B^c|A) = 0.001$

$P(B|A^c) = 0.0005$ $\Rightarrow P(B^c|A^c) = 0.9995$

2. Find the probability that the next email that makes it to my inbox will be a spam message. That is, find $P(A|B^c)$.

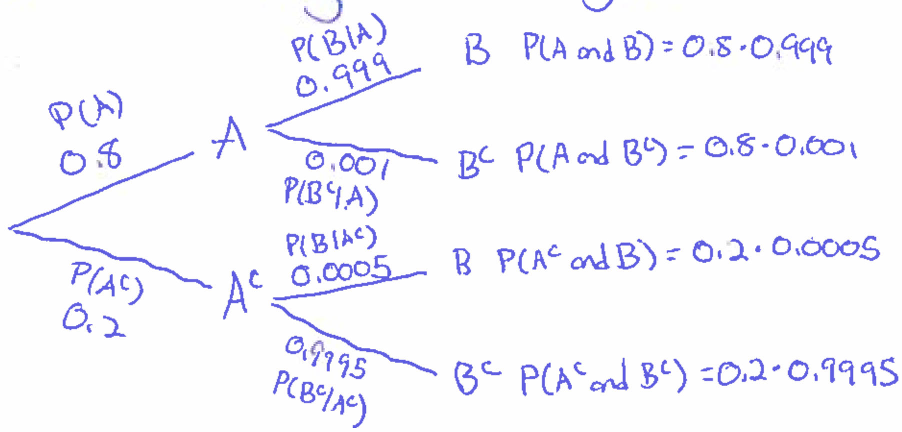
Use Bayes' Rule:

$$P(A|B^c) = \frac{P(B^c|A)\cdot P(A)}{P(B^c|A)\cdot P(A) + P(B^c|A^c)\cdot P(A^c)}$$

$$= \frac{0.001\cdot 0.8}{0.001\cdot 0.8 + 0.9995\cdot 0.2}$$

$$= 0.004$$

Here's a Probability Tree Diagram for this question:



Unsafe Food (Adapted from SDM4 14.39)

Early in 2010, *Consumer Reports* published the results of an extensive investigation of broiler chickens purchased from food stores in 23 states. Tests for bacteria in the meat showed that 9% of the chickens were contaminated with both campylobacter and salmonella, an additional 62% were contaminated with campylobacter but not salmonella, and another 14% were contaminated with salmonella.

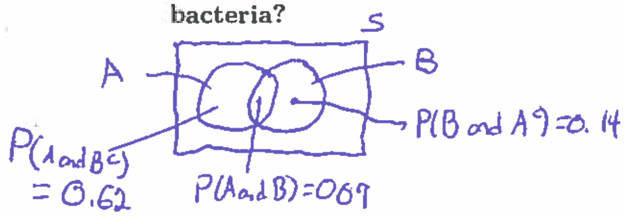
$P(A \text{ and } B) = 0.09$

$P(A \text{ and } B^c) = 0.62$

$P(B \text{ and } A^c) = 0.14$

Let A be the event that a randomly selected chicken is infected with campylobacter, and B be the event that a randomly selected chicken is infected with salmonella.

1. Whats' the probability that a tested chicken was not contaminated with either kind of bacteria?



From the Venn diagram at the left, we can see that $P(A \text{ or } B) = P(A \text{ and } B^c) + P(A \text{ and } B) + P(B \text{ and } A^c)$

$= 0.62 + 0.09 + 0.14$

$= \boxed{0.85}$

2. What's the probability that a randomly selected chicken is infected with campylobacter?

From the Venn diagram above we can see that

$P(A) = P(A \text{ and } B^c) + P(A \text{ and } B)$

$= 0.62 + 0.09$

$= \boxed{0.71}$

3. Given that a chicken is contaminated with salmonella, what's the probability that it is also contaminated with campylobacter?

By definition, $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

We can use the same idea as in part 2 to get $P(B) = P(A \text{ and } B) + P(B \text{ and } A^c)$

$= 0.09 + 0.14$

$= \boxed{0.23}$

Plugging into the above, we get $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{0.09}{0.23} = 0.39$

4. Are the events of being contaminated with campylobacter and being contaminated with salmonella independent?

To check for independence we need to check any one of the following 3 equations: $P(A) = P(A|B)$ OR $P(B) = P(B|A)$ OR $P(A \text{ and } B) = P(A) \cdot P(B)$.

From part 2, we have $P(A) = 0.71$, and from part 3, $P(A|B) = 0.39$.

Since $P(A) \neq P(A|B)$, the events A and B are not independent.

5. Are the events of being contaminated with campylobacter and being contaminated with salmonella disjoint?

The events A and B are not disjoint since
P(A and B) = 0.09, which is not equal to 0.